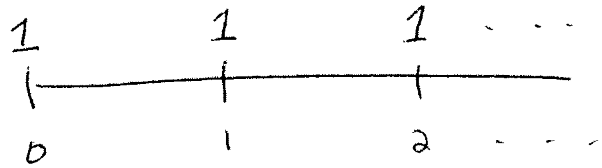
14/ May 2000
Course 2 Exam) Same aeir $\Rightarrow$ Same seir

and same baeir

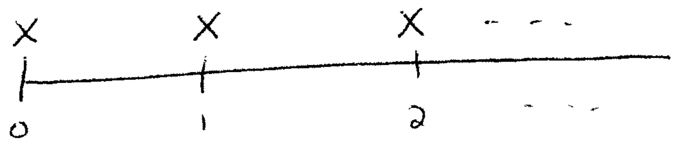
semiannual periods



i – seir

$$PV = 1 + \frac{1}{i} = 20 \Rightarrow i = \frac{1}{19}$$

bi-annual periods



j = baeir

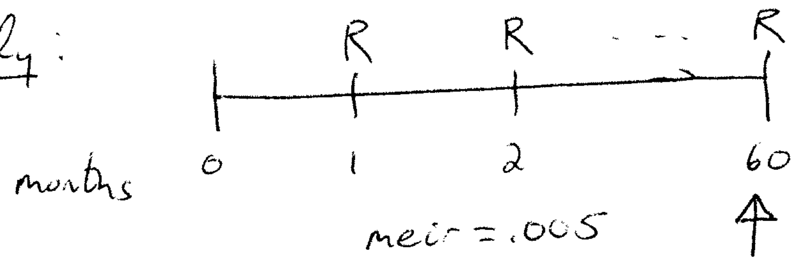
$$PV = 20 = X + \frac{X}{j}$$

$$1 + j = (1+i)^4 \Rightarrow j = \left(1 + \frac{1}{19}\right)^4 - 1$$

$$\therefore X \doteq 3.7$$

39/ May 2000 Course 2 Exam) Let R denote the amount of the monthly payments.

Sally:



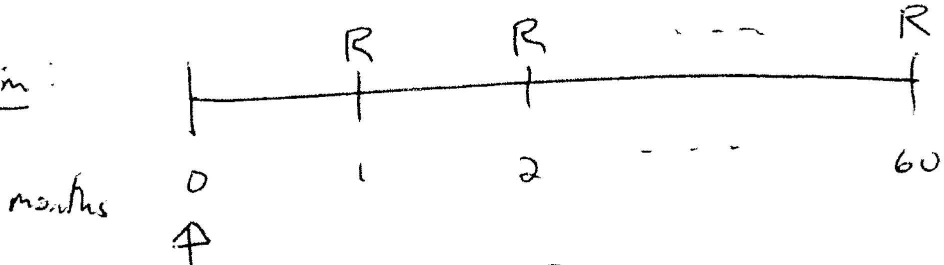
$$AV = R \cdot s_{\overline{60}|.005}$$

Since Sally's yield over the 5-year period is $i^{(2)} = .0745$, then $AV = 10000\left(1+\frac{.0745}{2}\right)^{10}$

$$\therefore \quad R\, s_{\overline{60}|.005} = 10000\left(1+\frac{.0745}{2}\right)^{10}$$

$$\Longrightarrow R \doteq 206.61675$$

Tim:



$$PV = 10000 = R\, a_{\overline{60}|j} \Longrightarrow j = 0.73338$$

meir

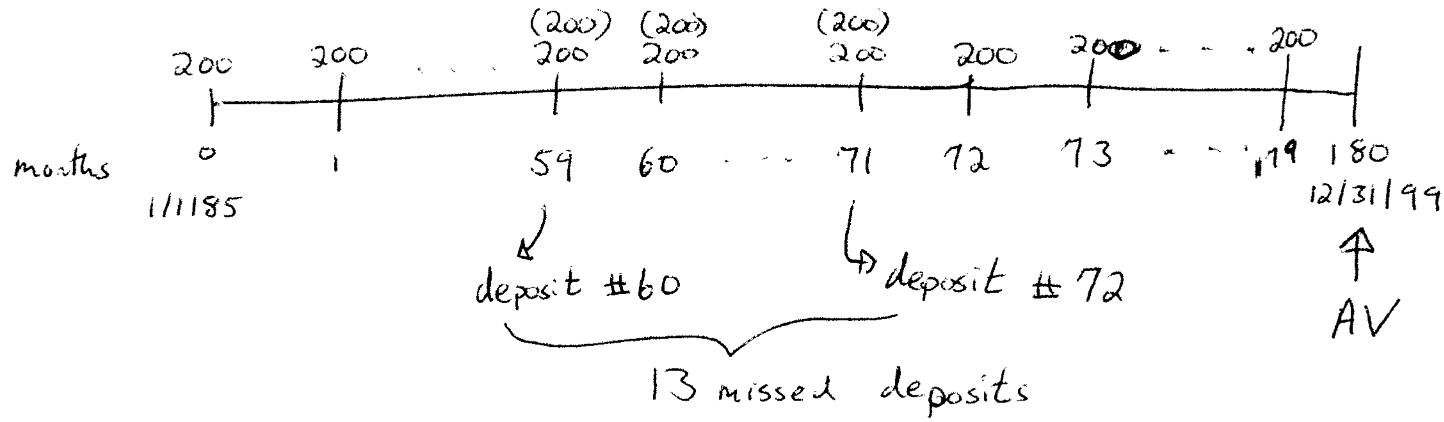
$\Longrightarrow$ nominal rate, compounded monthly, is

$$i^{(12)} = 12j \doteq 8.80\%$$

47/ May 2000 Course 2 Exam

.005 meir

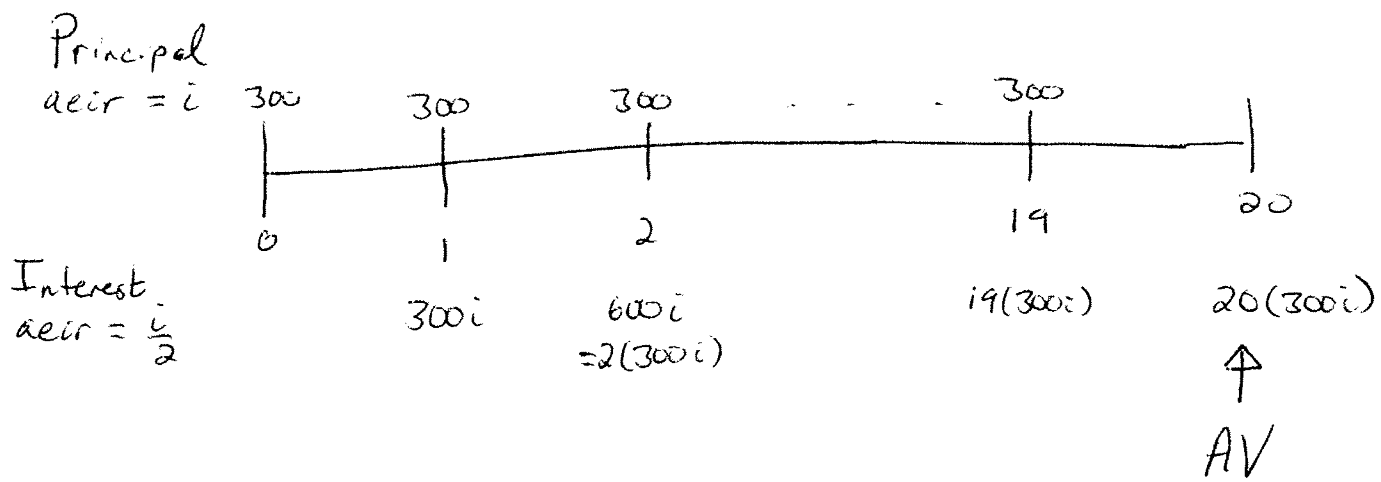
put in 200, take out 200
net = 0.



$$AV = 200\,\ddot{s}_{\overline{180}|.005} - \underbrace{200\,\ddot{s}_{\overline{13}|.005}}_{\text{time 72 value}} \cdot (1.005)^{180-72}$$

$\therefore AV \doteq 53{,}840$

9/ Nov. 2000
Course 2 Exam



$$AV = 300(20) + 300i \cdot (Is)_{\overline{20|}\, i/2}$$

$$(Is)_{\overline{n|}j} = (Ia)_{\overline{n|}j}(1+j)^n = \frac{\ddot{a}_{\overline{n|}} - nv^n}{j}(1+j)^n$$ distribute

$$\Longrightarrow (Is)_{\overline{n|}j} = \frac{\ddot{s}_{\overline{n|}} - n}{j}$$

$$\therefore AV = 6000 + 300i \cdot \frac{\ddot{s}_{\overline{20|}\, i/2} - 20}{i/2}$$

$$= 6000 + 600(\ddot{s}_{\overline{20|}\, i/2} - 20)$$

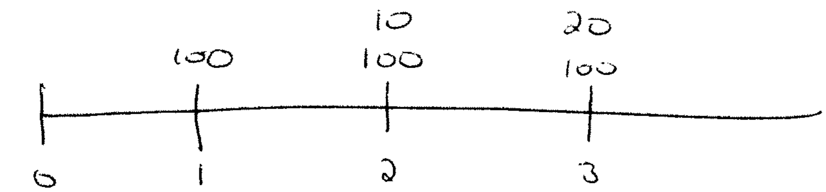
Since the yield over the 20 year period is .08 annual effective, then $AV = 300\,\ddot{s}_{\overline{20|}.08} \doteq 14826.87643$

$$\therefore 14826.87643 = 6000 + 600(\ddot{s}_{\overline{20|}\, i/2} - 20)$$

$$\Longrightarrow i \doteq 10\%$$

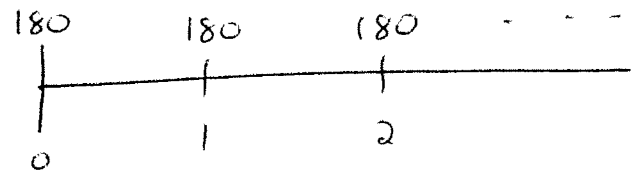
20 / Nov. 2000 Course 2 Exam

Sandy:



$$PV = \frac{P}{i} + \frac{Q}{i^2} = \frac{100}{i} + \frac{10}{i^2}$$

Danny:



$$PV = 180 + \frac{180}{i}$$

$$\therefore \left[\frac{100}{i} + \frac{10}{i^2} = 180 + \frac{180}{i}\right] * i^2$$

$$\Rightarrow 100i + 10 = 180i^2 + 180i$$

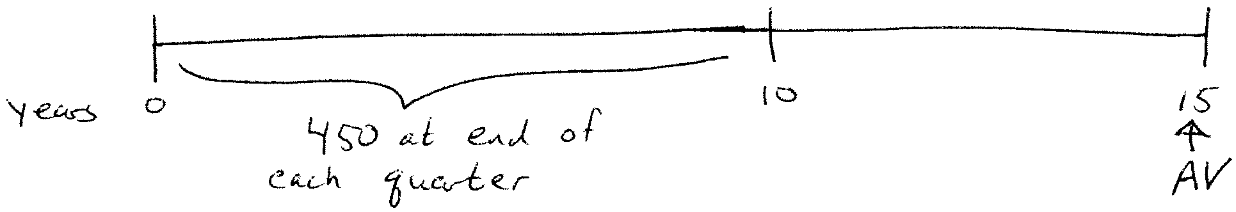
$$\Rightarrow 180i^2 + 80i - 10 = 0$$

$$18i^2 + 8i - 1 = 0$$

$$i = \frac{-8 \pm \sqrt{64 - 4(18)(-1)}}{2(18)} \doteq 10.29\%$$

22/ Nov. 2000 Course 2 Exam) aeir = .07

Determine the amount Jerry has at the end of 15 years:



Note: Using upper 4 notation, we get

$$AV = 1800\, s^{(4)}_{\overline{10}|.07} \cdot (1.07)^5 = 1800\, \frac{(1.07)^{10} - 1}{i^{(4)}}\, (1.07)^5$$

$$\left(1 + \frac{i^{(4)}}{4}\right)^4 = 1.07 \Rightarrow i^{(4)} = 4\left[(1.07)^{1/4} - 1\right]$$

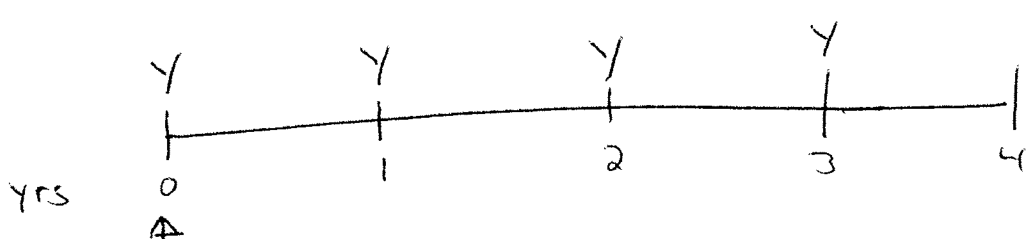
$$\therefore AV = 35783.62809$$

Without upper 4 notation, we get

$$AV = 450\, s_{\overline{40}|j}\, (1.07)^5 \qquad j = \text{qeir} = (1.07)^{1/4} - 1$$

$$\Rightarrow AV = 35783.62809$$

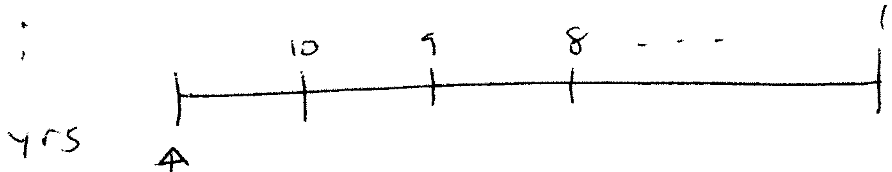
Now we have, for the annual payments of Y,



$$PV = 35783.62809 = Y\, \ddot{a}_{\overline{4}|.07} \Rightarrow Y \doteq 9873$$

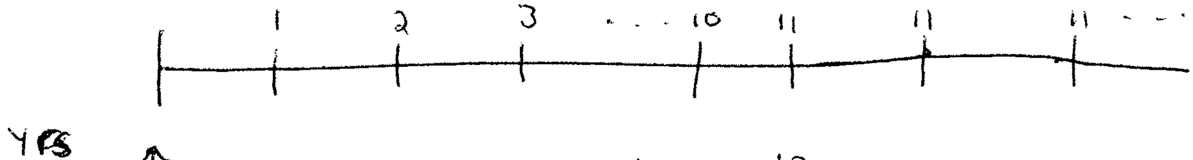
44 / (Nov. 2000 Course 2 Exam)

Annuity 1 :



$$PV_1 = (Da)_{\overline{10}|i}$$

Annuity 2 :



$$PV_2 = (Ia)_{\overline{10}|i} + \frac{11}{i} \cdot v^{10}$$

$$PV_2 = 2\,(PV_1)$$

$$\therefore \quad (Ia)_{\overline{10}|} + \frac{11}{i} v^{10} = 2 \cdot (Da)_{\overline{10}|}$$

$$\left[\frac{\ddot{a}_{\overline{10}|} - 10v^{10}}{i} + \frac{11v^{10}}{i} = 2\,\frac{10 - a_{\overline{10}|}}{i} \right] * i$$

$$\underbrace{\ddot{a}_{\overline{10}|} + v^{10}}_{} = 2\,(10 - a_{\overline{10}|})$$

$$\underline{\underline{\text{VEP}}}\ (1 + v + v^2 + \cdots + v^9) + v^{10} = 1 + (v + v^2 + \cdots + v^{10})$$

$$\underline{\underline{\text{VEP}}}\ 1 + a_{\overline{10}|}$$

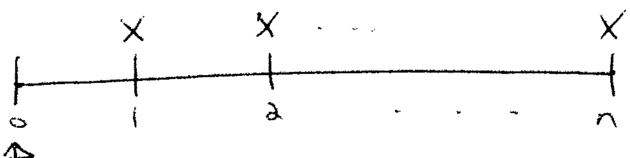
$$\therefore \quad 1 + a_{\overline{10}|} = 2(10 - a_{\overline{10}|}) \implies a_{\overline{10}|} = \frac{19}{3}$$

$$\implies i \doteq .093016$$

$$PV_1 = (Da)_{\overline{10}|i} = \frac{10 - a_{\overline{10}|}}{i} \doteq 39.4$$

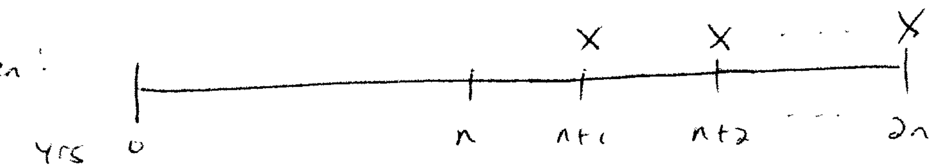
5/ May 2001 Course 2 Exam

Brian:



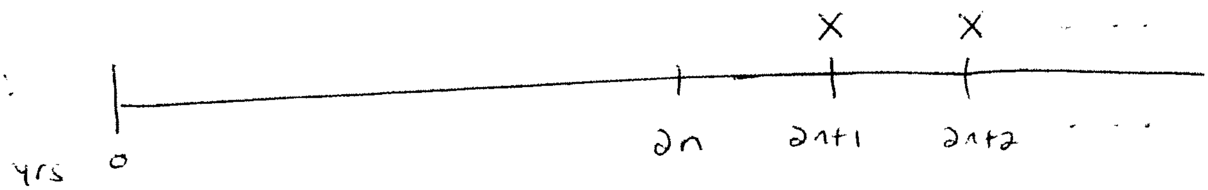
$PV = X a_{\overline{n}|}$

Colleen:



$PV = X a_{\overline{n}|} \cdot v^n$

Jeff:



$PV = \frac{X}{i} \cdot v^{2n}$

$$PV_B = .4 \cdot \frac{X}{i} = X a_{\overline{n}|} = X \cdot \frac{1 - v^n}{i} \implies v^n = .6$$

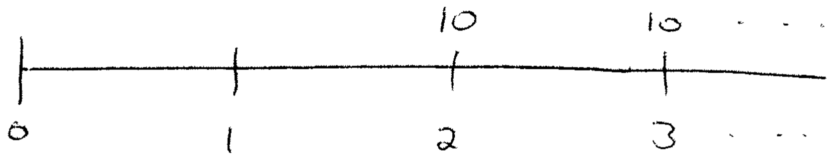
$$\therefore PV_J = v^{2n} \cdot \frac{X}{i} = (.6)^2 \cdot \frac{X}{i} = .36 \cdot \frac{X}{i}$$

= 36% of the present value of the original perpetuity

17 / (May 2001 Course 2 Exam)

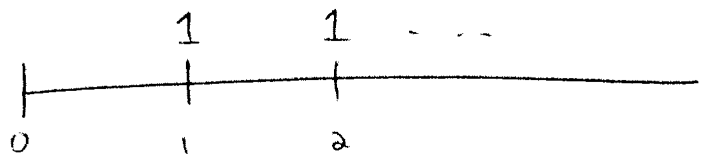
Same aeir $\Longrightarrow$ Same 3-year eir and same 4-month eir

3-year periods



$$PV = 32 = \frac{10}{j} \cdot v_j = \frac{10}{j(1+j)} \qquad j = \text{3-year eir}$$

4-month periods



$$PV = X = \frac{1}{k} \qquad k = \text{4-month eir}$$

$$32 = \frac{10}{j+j^2} \Longrightarrow 32j^2 + 32j - 10 = 0$$

$$\Longrightarrow j = \frac{-32 \overset{+}{\cancel{-}} \sqrt{32^2 - 4(32)(-10)}}{2(32)} = 0.25$$

$$(1+k)^9 = 1+j \Longrightarrow k = (1+j)^{1/9} - 1$$

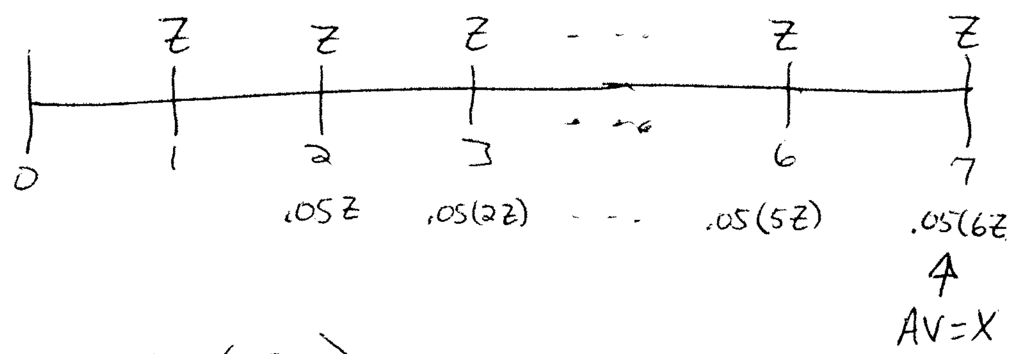
$$X = \frac{1}{k} \doteq 39.8$$

26 / May 2001
Course 2 Exam

Susan

Principal
aeir = .05

Interest
aeir = .06

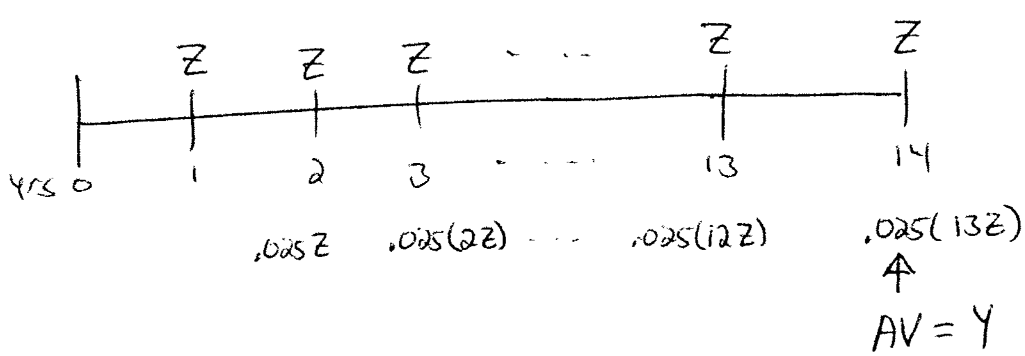


$$X = 7Z + .05Z \cdot (Is)_{\overline{6|}.06}$$
$$= Z\left[7 + .05(Is)_{\overline{6|}.06}\right]$$

Lori

Principal
aeir = .025

Interest
aeir = .03



$$Y = 14Z + .025Z \cdot (Is)_{\overline{13|}.03}$$
$$= Z\left[14 + .025(Is)_{\overline{13|}.03}\right]$$

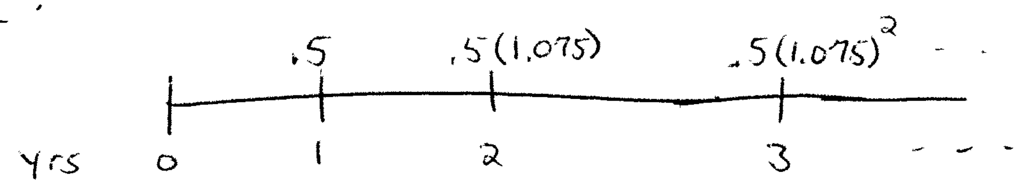
$$\therefore \quad \frac{Y}{X} = \frac{14 + .025(Is)_{\overline{13|}.03}}{7 + .05(Is)_{\overline{6|}.06}} \doteq 2.03$$

<u>Note:</u> $(Is)_{\overline{n|}} = \dfrac{\ddot{a}_{\overline{n|}} - nv^n}{i}(1+i)^n = \dfrac{\ddot{s}_{\overline{n|}} - n}{i}$

48 / May 2001 Course 2 Exam

Stock Price = PV(dividends)

Original:



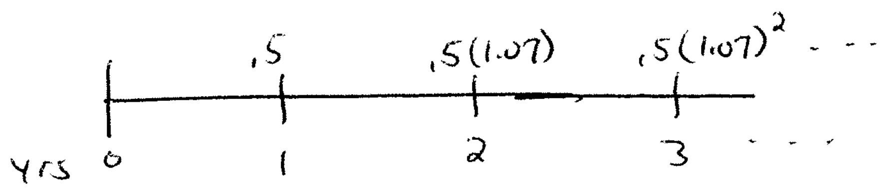
$$PV = 28.5 \overset{VEP}{=\!=} .5v + .5(1.075)v^2 + \cdots \quad \begin{pmatrix} \text{geometric} \\ \text{common ratio} \\ = 1.075v \end{pmatrix}$$

$$\therefore\ 28.5 = \frac{.5v}{1 - 1.075v}\left(\frac{1+i}{1+i}\right)$$

$$\Rightarrow 28.5 = \frac{.5}{(1+i) - 1.075} = \frac{.5}{i - .075}$$

$$\Rightarrow i \doteq .092544$$

Updated:



$$PV \overset{VEP}{=\!=} .5v + .5(1.07)v^2 + \cdots \quad \begin{pmatrix} \text{geometric:} \\ \text{common ratio} \\ = 1.07v \end{pmatrix}$$

$$= \frac{.5v}{1 - 1.07v} = \frac{.5}{i - .07} \doteq 22.2$$

50 / May 2001 Course 2 Exam

$$PV(i) = \frac{1}{.0725}$$

$$PV(ii) = a_{\overline{50|}j}$$

$$PV(iii) = a_{\overline{n|}(j-1)}$$

$$\Rightarrow a_{\overline{50|}j} = \frac{1}{.0725} \xRightarrow{TVM} j \doteq .07$$

$$\Rightarrow a_{\overline{50|}.07} = a_{\overline{n|}.06}$$

$$\Rightarrow n \doteq 30$$

5 / Nov. 2001 Course 2 Exam

		10	10	10	10	10	$10(1+.01K)$	$10(1+.01K)^2$ ...
Years	0	1	2	3	4	5	6	7 ...

$$PV = 167.50 = 10\,a_{\overline{5}|.092} + \underbrace{10(1+.01K)v^6 + 10(1+.01K)^2v^7 + \cdots}_{\substack{\text{geometric} \\ \text{common ratio} = (1+.01K)v}}$$

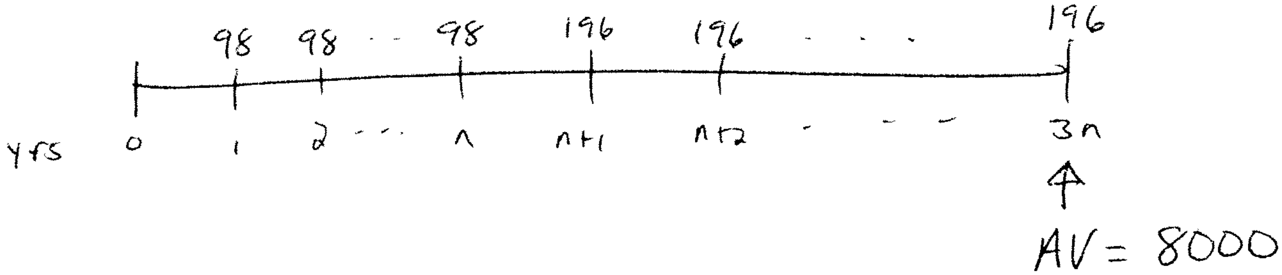
$$v = \frac{1}{1.092}$$

$$\therefore\ 167.50 = \underbrace{10\,a_{\overline{5}|.092}}_{\doteq_{\text{TVM}}\ 38.6955} + \frac{10(1+.01K)v^6}{1-(1+.01K)v}$$

$$\therefore\ \frac{10(1+.01K)v^6}{1-(1+.01K)v} \doteq 128.8045 \quad \left(\begin{array}{c}\text{Linear} \\ \text{equation} \\ \text{in } K\end{array}\right)$$

$$\Longrightarrow K \doteq 4.0$$

12 / Nov. 2001 Course 2 Exam



$$8000 = 98\, s_{\overline{n}|}\,(1+i)^{2n} + 196\, s_{\overline{2n}|}$$

$$= 98\,\frac{(1+i)^n - 1}{i}\,(1+i)^{2n} + 196\,\frac{(1+i)^{2n} - 1}{i}$$

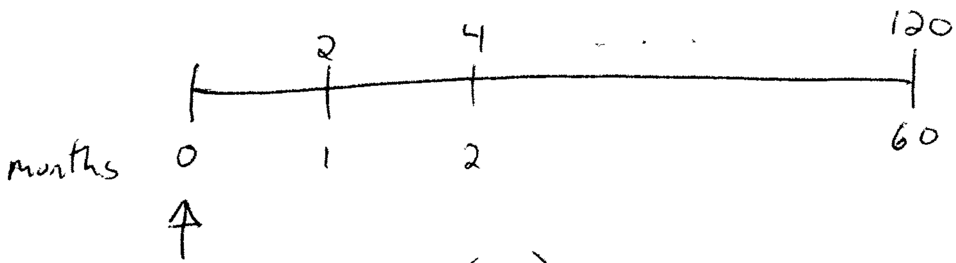
$$(1+i)^n = 2$$

$$\therefore\ 8000 = 98\,\frac{2-1}{i}\,(4) + 196\,\frac{4-1}{i}$$

$$\Rightarrow 8000 = \frac{980}{i} \Rightarrow i = 12.25\%$$

16 / Nov. 2001 Course 2 Exam

$$qeir = \frac{.09}{4} = .0225$$



$$PV = X = 2\,(Ia)_{\overline{60}|i} \qquad i = meir$$

$$(1+i)^3 = 1.0225 \implies i = (1.0225)^{1/3} - 1$$

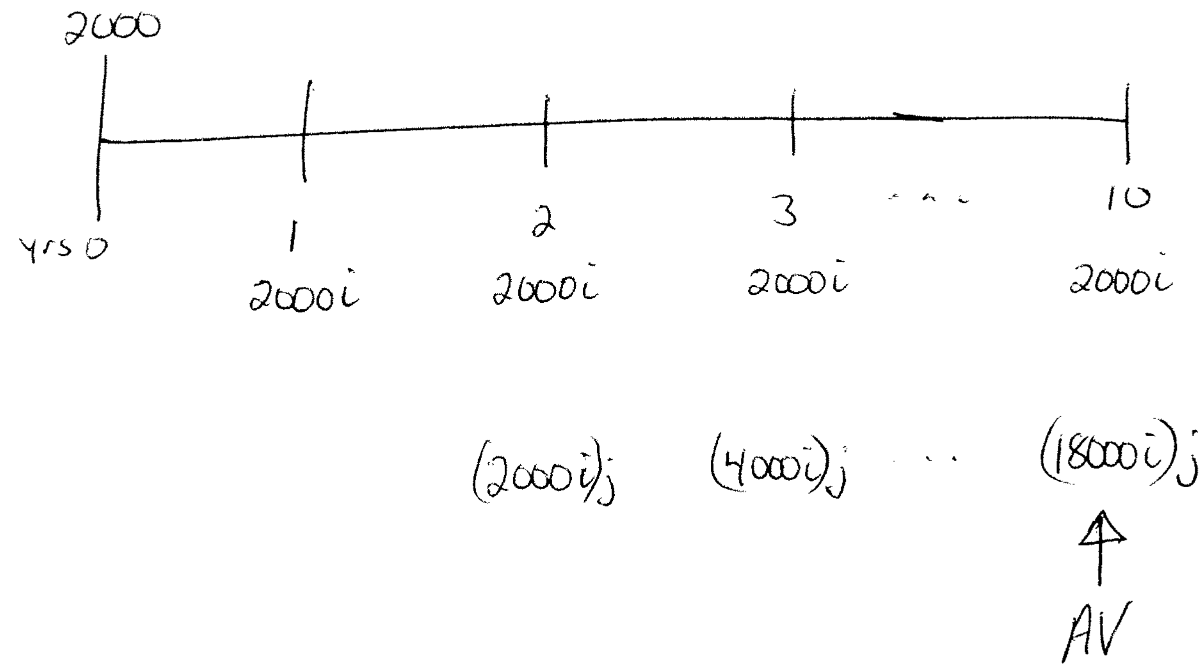
$$X = 2 \cdot \frac{\ddot{a}_{\overline{60}|} - 60v^{60}}{i} \doteq 2730$$

35/ Nov. 2001
Course 2 Exam

Principal
.02 qeir
i aeir

Interest
.0225 qeir
j aeir

Interest on
Interest
.07 aeir



$$AV = 2000 + 2000i(10) + (2000i)\cdot j \cdot (Is)_{\overline{9}|.07}$$

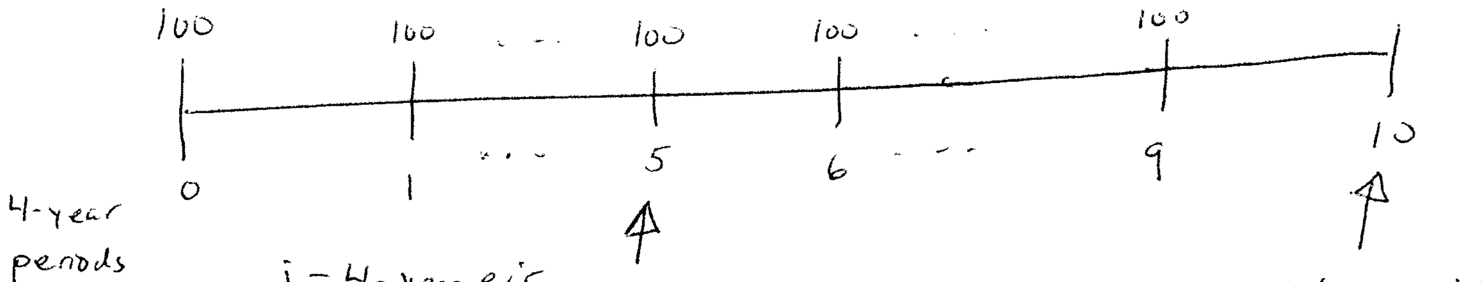
$$(1.02)^4 = 1 + i$$

$$(1.0225)^4 = 1 + j$$

$$(Is)_{\overline{9}|.07} = \frac{\ddot{s}_{\overline{9}|} - 9}{.07}$$

$$\therefore AV \doteq 4485$$

8 / (May 2003 Course 2 Exam)



$j = 4\text{-year eir}$

$AV_{20\text{ years}} = 100\,\ddot{s}_{\overline{5}|j}$

$AV_{40\text{ years}} = X$

$$AV_{40\text{ years}} = 100\,\ddot{s}_{\overline{10}|j} = 5 \cdot 100\,\ddot{s}_{\overline{5}|j}$$

$$\Longrightarrow \cancel{100}\left((1+j)^{10} - 1\right) = \cancel{500}\,5\left((1+j)^{5} - 1\right)$$

$$\Longrightarrow (1+j)^{10} - 5(1+j)^{5} + 4 = 0$$

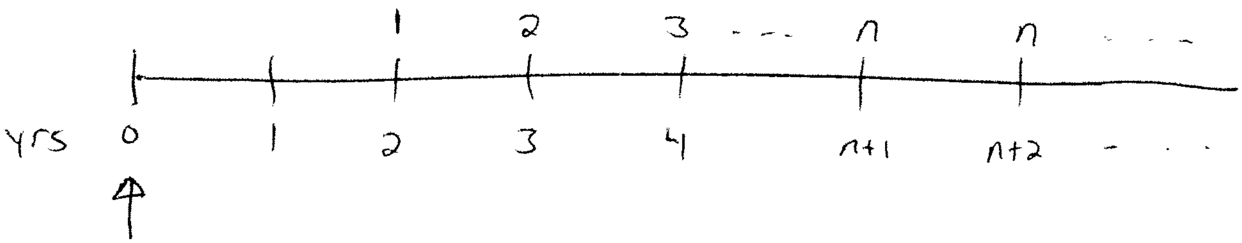
$$\Longrightarrow (1+j)^{5} = \frac{5 \pm \sqrt{25 - 4(1)(4)}}{2(1)} \Longrightarrow (1+j)^{5} = 4$$

$$\therefore\; j = 4^{1/5} - 1$$

$$\therefore\; X = 100\,\ddot{s}_{\overline{10}|j} \doteq 6195$$

22/ May 2003 Course 2 Exam

aeir = .105



$$PV = 77.1 = (Ia)_{\overline{n|}} \cdot v + \frac{n}{i} \cdot v^{n+1}$$

$$= \frac{\ddot{a}_{\overline{n|}} - n v^n}{i} \cdot v + \frac{n}{i} v^{n+1}$$

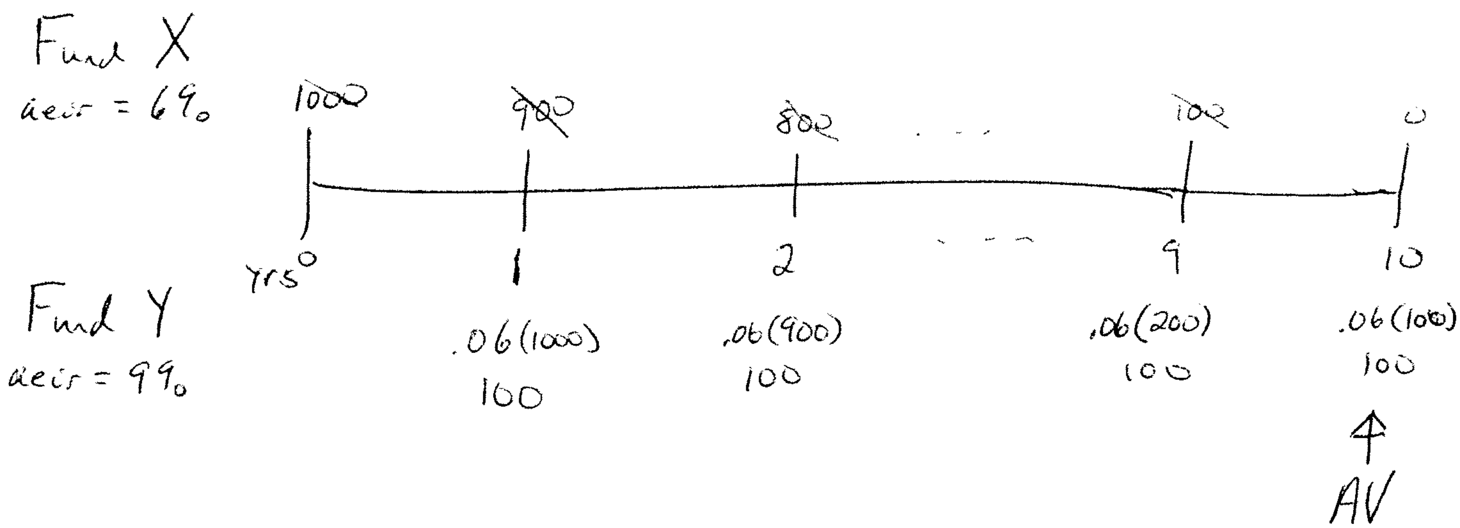
$$= \frac{\ddot{a}_{\overline{n|}} \cdot v - \cancel{n v^{n+1}} + \cancel{n v^{n+1}}}{i} = \frac{\ddot{a}_{\overline{n|}} \cdot v}{.105}$$

$$\ddot{a}_{\overline{n|}} \cdot v \overset{VEP}{=} (1 + v + v^2 + \cdots + v^{n-1}) \cdot v$$

$$= v + v^2 + v^3 + \cdots + v^n \overset{VEP}{=} a_{\overline{n|}}$$

$$\therefore (77.1)(.105) = a_{\overline{n|},.105} \xRightarrow{TVM} n \doteq 19$$

26/ May 2003 Course 2 Exam



$$AV = .06(100)(Ds)_{\overline{10|}.09} + 100\, s_{\overline{10|}.09}$$

$$(Ds)_{\overline{n|}} = (Da)_{\overline{n|}}(1+i)^n = \frac{n - a_{\overline{n|}}}{i}(1+i)^n$$

$$\therefore (Ds)_{\overline{n|}} = \frac{n(1+i)^n - s_{\overline{n|}}}{i}$$

$$\therefore AV \doteq 2085$$

33/ (May 2003 Course 2 Exam) $i = aeir$

$$PV(i) = 55\, a_{\overline{20|}}$$

		30	30 . . .	30	60	60 . . .	60	90	90 . . .	90
yrs	0	1	2 . . .	10	11	12 . . .	20	21	22 . . .	30

$$PV(ii) = 30\, a_{\overline{10|}} + 60\, a_{\overline{10|}} \cdot v^{10} + 90\, a_{\overline{10|}} \cdot v^{20}$$

$$PV(i) = 55\, a_{\overline{10|}} + 55\, a_{\overline{10|}}\, v^{10}$$

$$X = PV(i) = PV(ii)$$

$$\Longrightarrow \cancel{a_{\overline{10|}}}\,(55 + 55 v^{10}) = \cancel{a_{\overline{10|}}}\,(30 + 60 v^{10} + 90 v^{20})$$

$$\Longrightarrow 90 v^{20} + 5 v^{10} - 25 = 0$$

$$\Longrightarrow v^{10} = \frac{-5 \pm \sqrt{25 - 4(90)(-25)}}{2(90)} = \frac{1}{2}$$

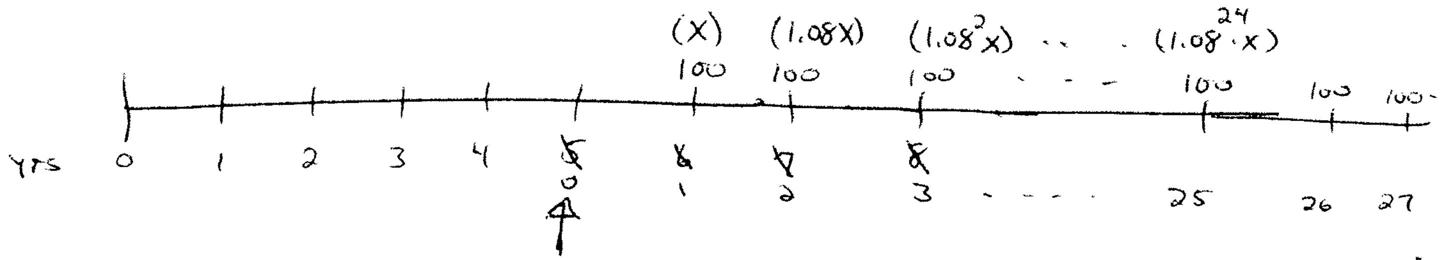
$$\Longrightarrow (1+i)^{10} = 2 \Longrightarrow i = 2^{1/10} - 1$$

$$X = 55\, a_{\overline{20|}i} \doteq 575$$

45 / (May 2003 Course 2 Exam)

aeir $= .08$

Immediately after the 5^{th} payment, we have



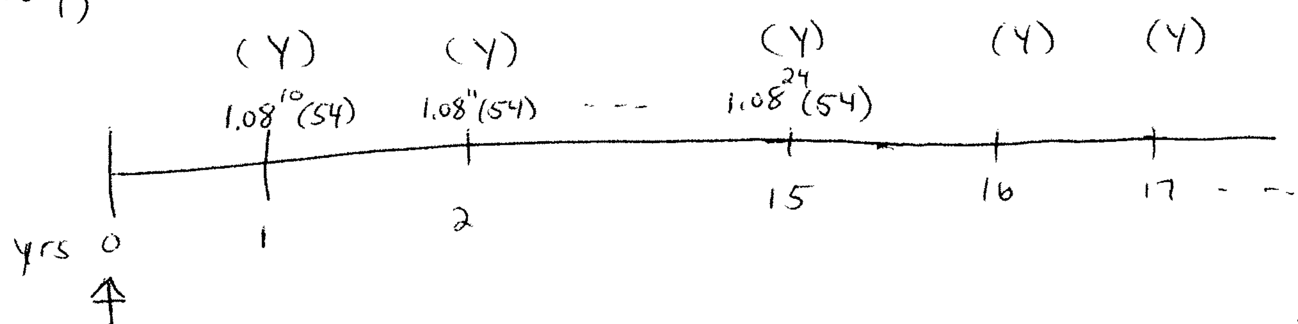
$$\frac{100}{i} \stackrel{VEP}{=} Xv + 1.08Xv^2 + \cdots \quad \left(\begin{array}{l}\text{Geometric}\\ \text{25 terms}\\ \text{common ratio}\\ = 1.08v\end{array}\right)$$

$i = .08;\ v = \frac{1}{1.08}$

$$\therefore \quad \frac{100}{.08} = Xv\Big(1 + \underbrace{1.08v}_{=1.08\cdot\frac{1}{1.08}=1} + \cdots \quad \left(\begin{array}{l}\text{25 terms; all}\\ \text{equal to 1}\end{array}\right)$$

$$\therefore \quad \frac{100}{.08} = X\left(\frac{1}{1.08}\right)(25) \Longrightarrow X = 54$$

Immediately after the 10^{th} payment of the 25-year annuity, we have



$$\frac{Y}{i} = (1.08)^{10}\cdot 54\cdot v + (1.08)^{11}\cdot 54\, v^2 + \cdots \quad \left(\begin{array}{l}\text{Geometric}\\ \text{15 terms; common}\\ \text{ratio} = 1.08v = 1\end{array}\right)$$

$$\therefore \quad \frac{Y}{.08} = \frac{(1.08)^{10}\cdot 54}{1.08}\Big(1 + 1 + \cdots \quad (15 \text{ terms})\Big)$$

$$\Longrightarrow Y = .08\,\frac{(1.08)^{10}\cdot 54}{1.08}(15) \doteq 130$$

1/May 2005 Course FM Exam

✓ (A) Distribute v^n to get $\frac{1-v^n}{i}$ (CRF)

✓ (B) (CRF)

✓ (C) (VEP)

✓ (D) $1-v = d = i \cdot v \Rightarrow v\left[\frac{1-v^n}{1-v}\right] = \frac{1-v^n}{i}$ (CRF)

x (E) $a_{\overline{n}|} = s_{\overline{n}|} \cdot v^n = s_{\overline{n}|}\left(\frac{1}{1+i}\right)^n = \frac{s_{\overline{n}|}}{(1+i)^n}$

$\therefore \quad \frac{s_{\overline{n}|}}{(1+i)^{n-1}} \neq a_{\overline{n}|}$

4/ (May 2005 Course FM Exam)

$PV(\text{Seth}) = X a_{\overline{n|}}$

$PV(\text{Susie}) = X a_{\overline{m|}} \cdot v^n$

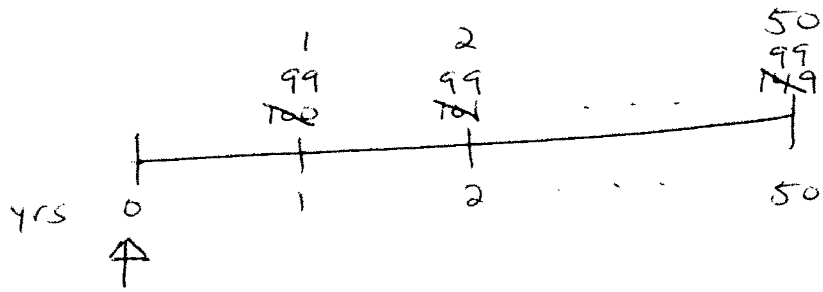
$PV(\text{Lori}) = \frac{X}{i} \cdot v^{n+m}$

$\therefore\ PV(\text{Seth}) - PV(\text{Susan})$

$= X a_{\overline{n|}} - X a_{\overline{m|}} v^n = X\left(a_{\overline{n|}} - a_{\overline{m|}} v^n\right)$

9/ (May 2005 Course FM Exam)

a e i r = .09



$\Delta = 1$

$$PV = X = 99\, a_{\overline{50}|.09} + (Ia)_{\overline{50}|.09}$$

$$= 99\, a_{\overline{50}|.09} + \frac{\ddot{a}_{\overline{50}|.09} - 50v^{50}}{.09}$$

$$\therefore\ X \doteq 1210$$

12 / (May 2005 Course FM Exam)

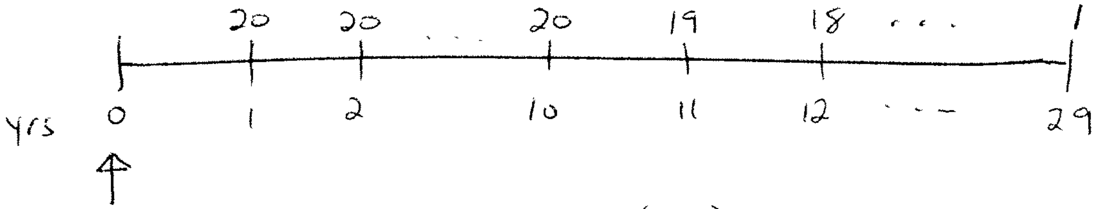
× I. is true for level annual perpetuity immediates.

So I. is not true generally

✓ II. is true for all perpetuities

× III. is true for the same case as I. above, but is not true generally.

14/ (May 2005 Course FM Exam) aeir = .06

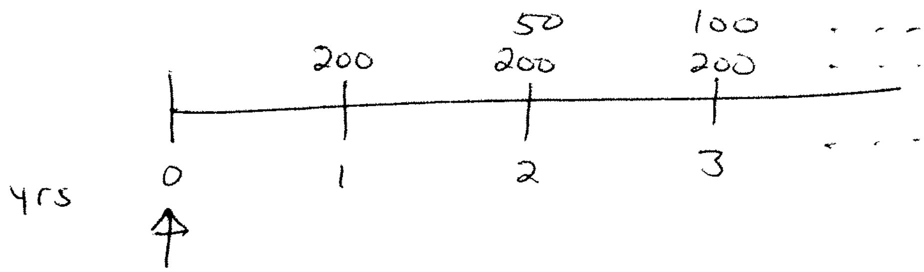


$$PV = X = 20\, a_{\overline{10}|.06} + (Da)_{\overline{19}|.06} \cdot v^{10}_{.06}$$

$$= 20\, a_{\overline{10}|.06} + \frac{19 - a_{\overline{19}|.06}}{.06} \cdot v^{10}_{.06}$$

$$\therefore \quad X \doteq 220$$

17 / May 2005 Course FM Exam

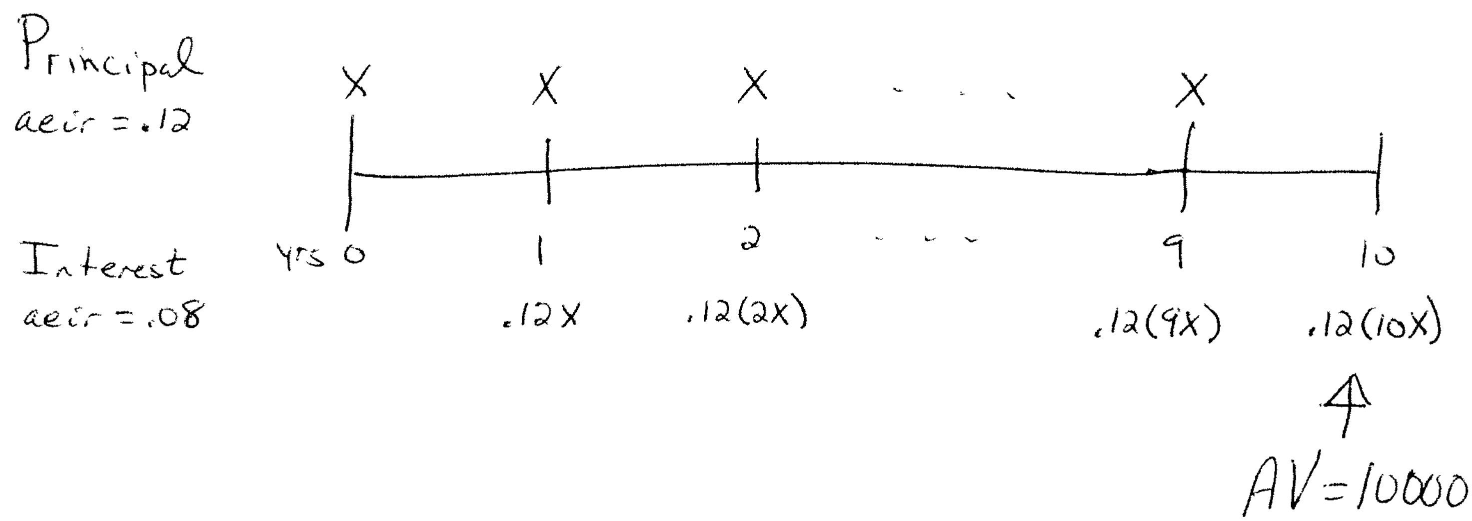


$$\left[PV = 46530 = \frac{200}{i} + \frac{50}{i^2} \right] * i^2$$

$$46530 i^2 - 200 i - 50 = 0$$

$$\Rightarrow i = \frac{200 \pm \sqrt{(-200)^2 - 4(46530)(-50)}}{2(46530)} \doteq .035$$

20 / May 2005 Course FM Exam) Let X = amount of each deposit



$$AV = 10X + .12X\,(Is)_{\overline{10}|.08} = 10000$$

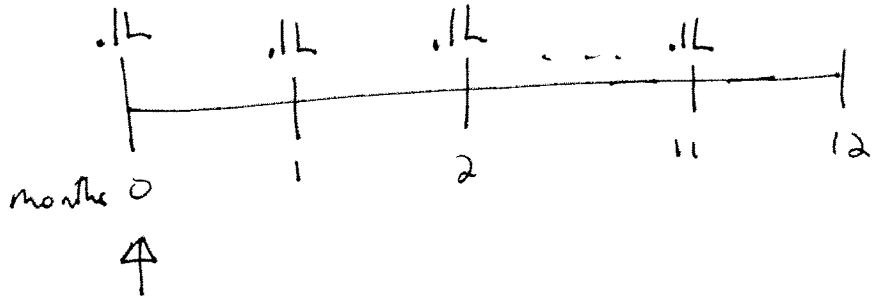
$$\Longrightarrow X = \frac{10000}{10 + .12\,(Is)_{\overline{10}|.08}} \doteq 541$$

Note: $(Is)_{\overline{10}|.08} = \dfrac{\ddot{s}_{\overline{10}|.08} - 10}{.08}$

21/ May 2005 Course FM Exam

Let $L =$ total cost

Then payments are $\frac{L}{10} = .1L$



$$PV = L = .1L\, \ddot{a}_{\overline{12}|m} \qquad m = \text{meir}$$

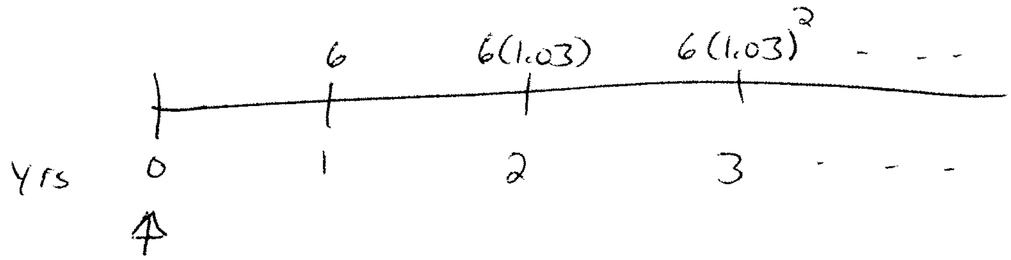
$$\Longrightarrow \ddot{a}_{\overline{12}|m} = 10 \Longrightarrow m \overset{TVM}{=} .035031$$

$$i = \text{aeir} \Longrightarrow 1+i = (1+m)^{12}$$

$$\Longrightarrow i \doteq 51.2\%$$

23 / (May 2005 Course FM Exam)

Stock Price = PV(dividends)



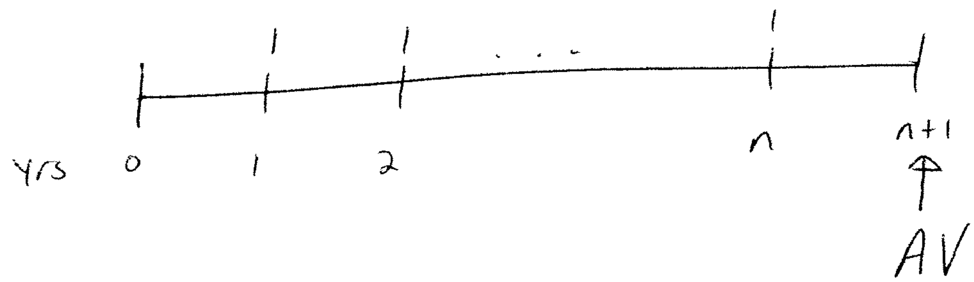
$$PV = 75 \overset{VEP}{=} 6v + 6(1.03)v^2 + \cdots$$
$\begin{pmatrix}\text{Geometric} \\ \text{Common ratio} \\ = 1.03v\end{pmatrix}$

$$\therefore 75 = \frac{6v}{1 - 1.03v} \cdot \left(\frac{1+i}{1+i}\right)$$

$$\Longrightarrow 75 = \frac{6}{(1+i) - 1.03} = \frac{6}{i - .03}$$

$$\Longrightarrow i = 11\%$$

24 / May 2005 Course FM Exam



$$AV = 13.776 = \ddot{s}_{\overline{n}|} = \frac{(1+i)^n - 1}{i}(1+i)$$

$$\therefore 13.776 = \frac{2.476 - 1}{i}(1+i)$$

$$\Rightarrow i = .12$$

$$\therefore (1.12)^n = 2.476$$

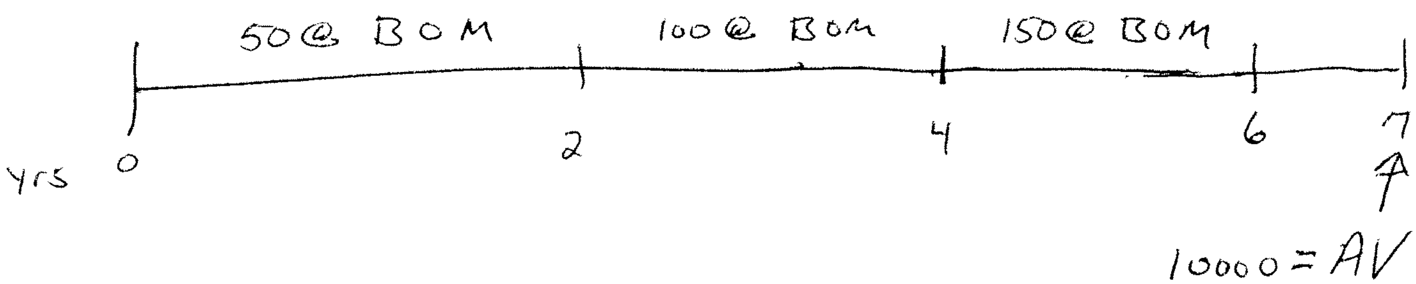
$$\Rightarrow n \doteq 8$$

3/ Nov. 2005 Exam FM

i = aeir

j = meir

BOM – Beginning of Month



$$AV = 50\,\ddot{s}_{\overline{24}|j}(1+i)^5 + 100\,\ddot{s}_{\overline{24}|j}(1+i)^3 + 150\,\ddot{s}_{\overline{24}|j}(1+i)$$

$$\therefore\ \cancel{50}\,\ddot{s}_{\overline{24}|j}(1+i)\left[(1+i)^4 + 2(1+i)^2 + 3\right] = \cancel{10000}\ 200$$

× (A) uses $\ddot{s}_{\overline{24}|i}$ (should be j)

× (B) No

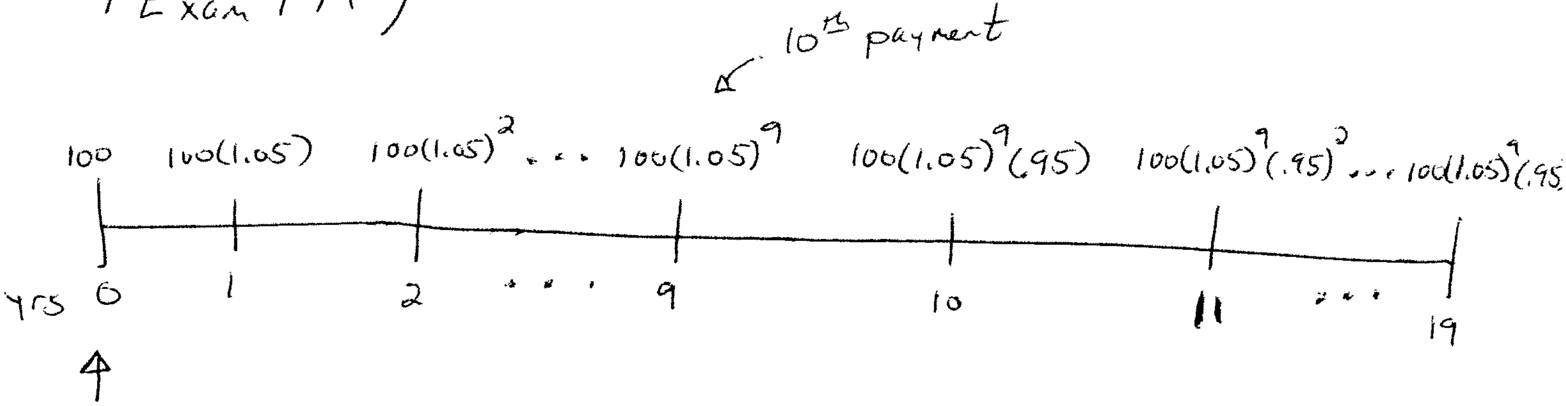
✓ (C) Yes

× (D) uses $s_{\overline{24}|j}$ (should be $\ddot{s}$)

× (E) No

8/Nov. 2005 Exam FM

$aeir = .07$



$PV \overset{VEP}{=} 100 + 100(1.05)v + \cdots + 100(1.05)^9 v^9$ (1st 10 payments; Geometric; common ratio $= 1.05v$)

$+ 100(1.05)^9(.95)v^{10} + 100(1.05)^9(.95)^2 v^{11} + \cdots$ (2nd 10 payments; Geometric; common ratio $= .95v$)

$= 100\left(1 + \frac{1.05}{1.07} + \cdots \text{ (10 terms)}\right)$

$+ 100(1.05)^9(.95)\left(\frac{1}{1.07}\right)^{10}\left[1 + \frac{.95}{1.07} + \cdots \text{ (10 terms)}\right)$

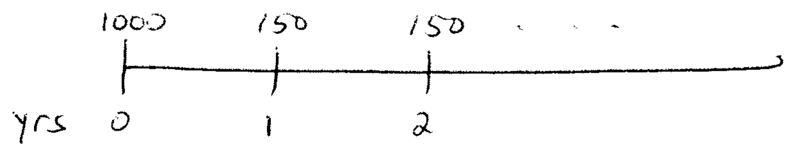
$\therefore PV = 100\, \ddot{a}_{\overline{10}|\left(\frac{1.07}{1.05}-1\right)}$

$+ 100(1.05)^9(.95)\frac{1}{(1.07)^{10}} \cdot \ddot{a}_{\overline{10}|\left(\frac{1.07}{.95}-1\right)}$

$\therefore PV \doteq 1385$

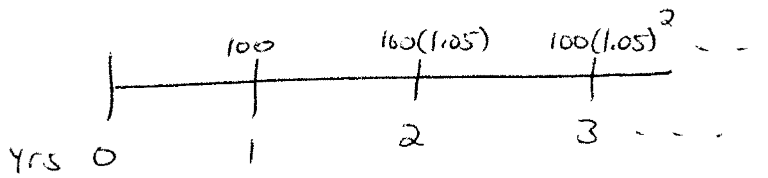
9/ Nov. 2005 Exam FM) $i = \text{aeir}$ (yield rate)

Deposits:



$$PV = 1000 + \frac{150}{i}$$

Payments:



$$PV \overset{VEP}{=\!=} 100v + 100(1.05)v^2 + \cdots \quad \left(\begin{array}{c}\text{Geometric} \\ \text{common ratio} = 1.05v\end{array}\right)$$

$$= \frac{100v}{1 - 1.05v} \cdot \left(\frac{1+i}{1+i}\right) = \frac{100}{(1+i) - 1.05} = \frac{100}{i - .05}$$

$$\therefore \left[1000 + \frac{150}{i} = \frac{100}{i - .05}\right] * i(i - .05)$$

$$1000(i^2 - .05i) + 150(i - .05) = 100i$$

$$1000i^2 - 50i + 150i - 7.5 = 100i$$

$$1000i^2 = 7.5$$

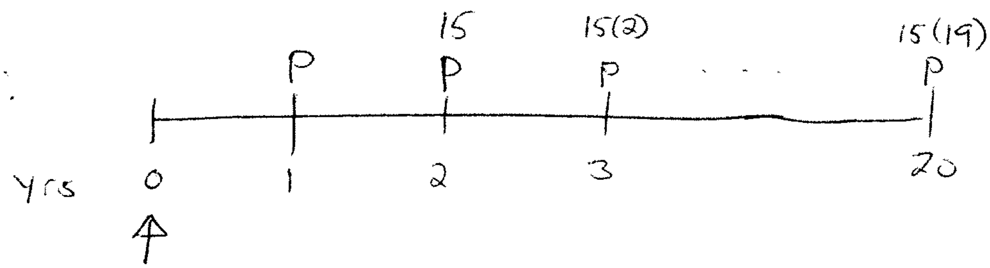
$$\Longrightarrow i \doteq 8.7\%$$

12 / Nov. 2005 Exam FM

$i = \text{aeir}$

Megan: $3250 = \dfrac{130}{i} \implies i = .04$

Chris:



$$PV = 3250 = P\, a_{\overline{20}|i} + 15\,(Ia)_{\overline{19}|} \cdot v_{.04}$$

$$(Ia)_{\overline{19}|} = \frac{\ddot{a}_{\overline{19}|.04} - 19\, v_{.04}^{19}}{.04}$$

$$\implies P \doteq 116$$

13 / (Nov. 2005 Exam FM)

$j =$ qeir

$$10000 = 400\, a_{\overline{40}|j}$$

$$\Longrightarrow\ j \overset{\text{TVM}}{=} .025244$$

$i =$ meir $\Longrightarrow\ (1+i)^3 = 1 + j$

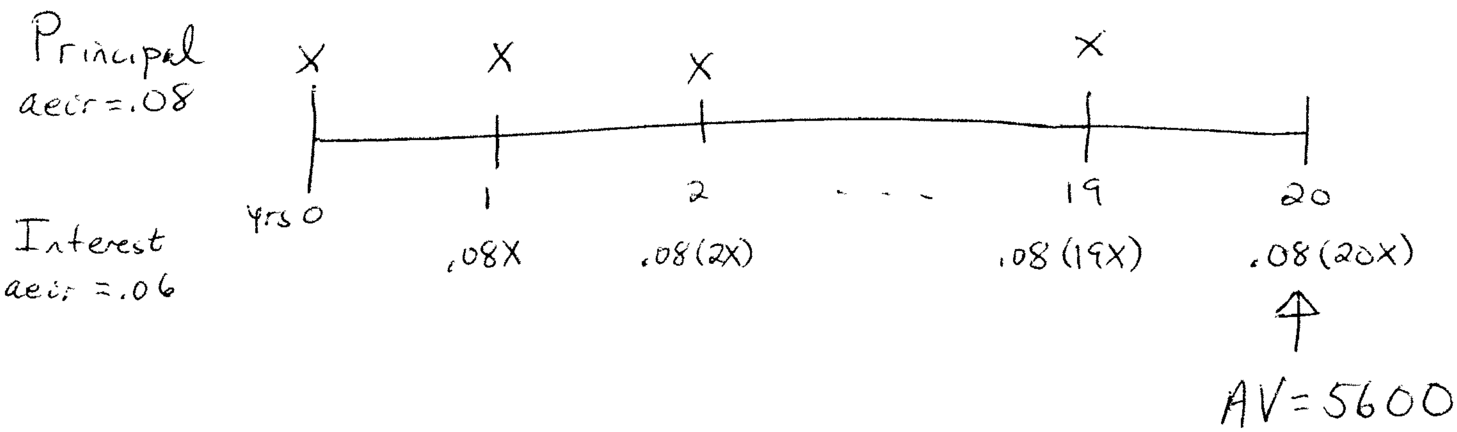
$$\Longrightarrow\ i = (1+j)^{1/3} - 1$$

$$\therefore\ i^{(12)} = 12 i = 12\left[(1+j)^{1/3} - 1\right]$$

$$i^{(12)} \doteq 10.0\,\%$$

14

Nov. 2005
Exam FM



$$AV = 20X + .08X(Is)_{\overline{20}|.06} = 5600$$

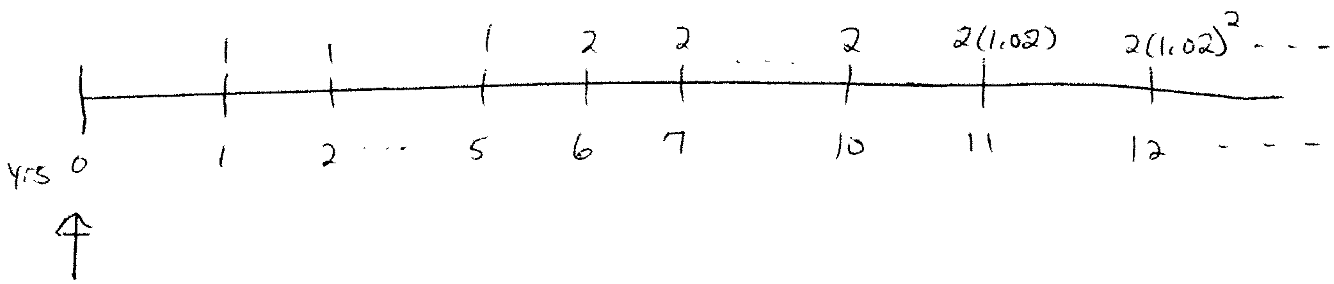
$$\Rightarrow X = \frac{5600}{20 + .08\,(Is)_{\overline{20}|.06}} \doteq 123.56$$

$$(Is)_{\overline{20}|.06} = \frac{\ddot{s}_{\overline{20}|.06} - 20}{.06}$$

20 / Nov. 2005 Exam FM

Stock Price = PV (dividends)

aeir = .06



$$PV = 2\,a_{\overline{10}|} - a_{\overline{5}|} + \underbrace{2(1.02)v^{11} + 2(1.02)^2 v^{12} + \cdots}_{\text{VEP}}$$

(Geometric; common ratio equals $1.02\,v = \frac{1.02}{1.06}$)

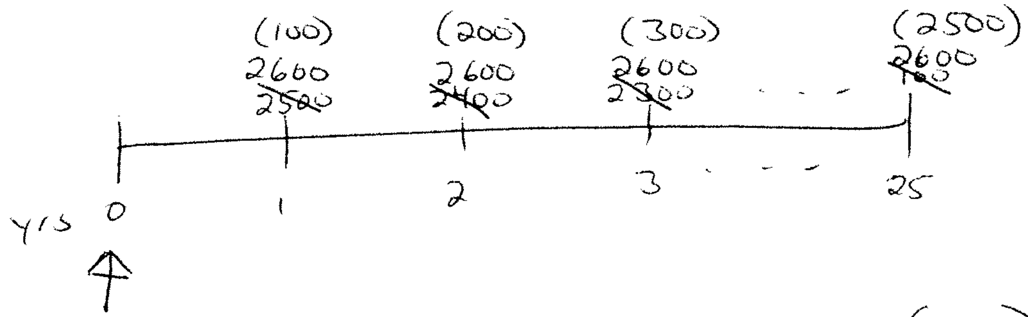
$$\therefore\ PV = 2\,a_{\overline{10}|.06} - a_{\overline{5}|.06} + \frac{2(1.02)\,v_{.06}^{11}}{1 - \frac{1.02}{1.06}}$$

$$\doteq 39$$

23 / Nov. 2005 Exam FM

aeir = 10%

$\Delta = 100$



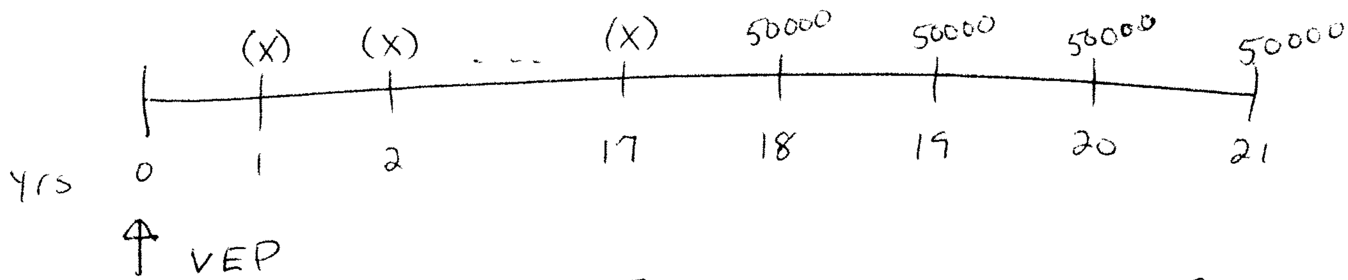
$$PV = X = 2600\, a_{\overline{25}|.10} - 100 (Ia)_{\overline{25}|.10}$$

$$= 2600\, a_{\overline{25}|.10} - 100\, \frac{\ddot{a}_{\overline{25}|.10} - 25 v_{.1}^{25}}{.1}$$

$$\therefore X \doteq 15923$$

49 / Exam FM Sample Questions

$aeir = .05$



$$PV = Xv + Xv^2 + \cdots + Xv^{17} = 50000v^{18} + 50000v^{19} + 50000v^{20} + 50000v^{21}$$

All v's use $i = .05$.

$$\therefore X\left(v + v^2 + \cdots + v^{17}\right) = 50000\left(v^{18} + v^{19} + v^{20} + v^{21}\right)$$

LHS = Left Hand Side RHS = Right Hand Side

× (A) No (val date of LHS is $t=0$; val date of RHS is $t=17$)

× (B) Change val date to $t=18$. Almost, but LHS only has 16 payments.

× (C) No (val date of LHS is $t=17$?; LHS has 18 payments)

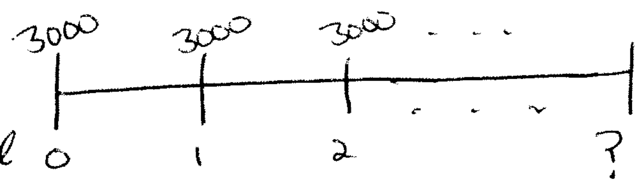
✓ (D) Yes, using val date at $t=18$. Multiply the PV equation above by $(1.05)^{18}$.

× (E) No, LHS has 18 payments

84/ Exam FM Sample Questions) First determine the amount accumulated after 20 years.

$$AV = 1000\,\ddot{s}_{\overline{20}|.0816}$$

Then 20 years later we have the following timeline:



semi-annual periods seir = .04

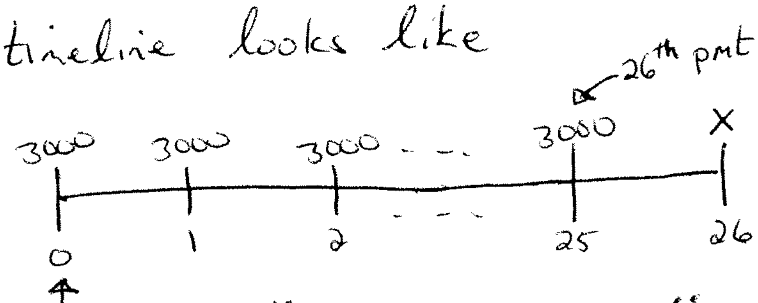
? we don't know how many payments

We have $PV = 1000\,\ddot{s}_{\overline{20}|.0816}$, being the AV from above.

$$\text{Solving } 1000\,\ddot{s}_{\overline{20}|.0816} = 3000\,\ddot{a}_{\overline{n}|.04} \xRightarrow{TVM} n = 26+$$

$\therefore$ there are 26 full payments and a smaller payment one period after the last full payment.

$\therefore$ the timeline looks like

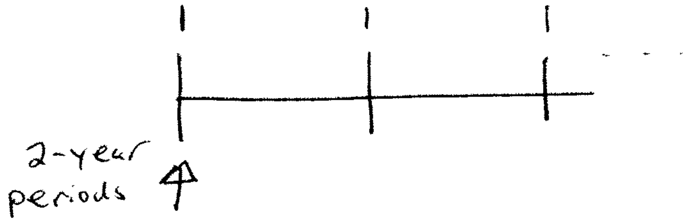


semiannual periods seir = .04

$$PV = 1000\,\ddot{s}_{\overline{20}|.0816} = 3000\,\ddot{a}_{\overline{26}|.04} + X\,v_{.04}^{26}$$

$$\Longrightarrow X = 1430.36$$

85 / Exam FM Sample Questions) For the first perpetuity, we have

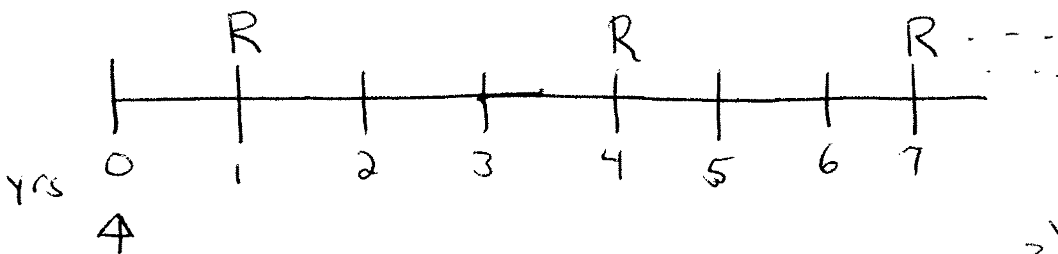


j = 2-year eir (baeir)

$$PV = 7.21 = 1 + \frac{1}{j} \Longrightarrow j = \frac{1}{6.21}$$

$$1 + j = (1+i)^2 \Longrightarrow i \doteq .0775 \Longrightarrow i + .01 \doteq .0875$$

$\therefore$ for the second perpetuity we have

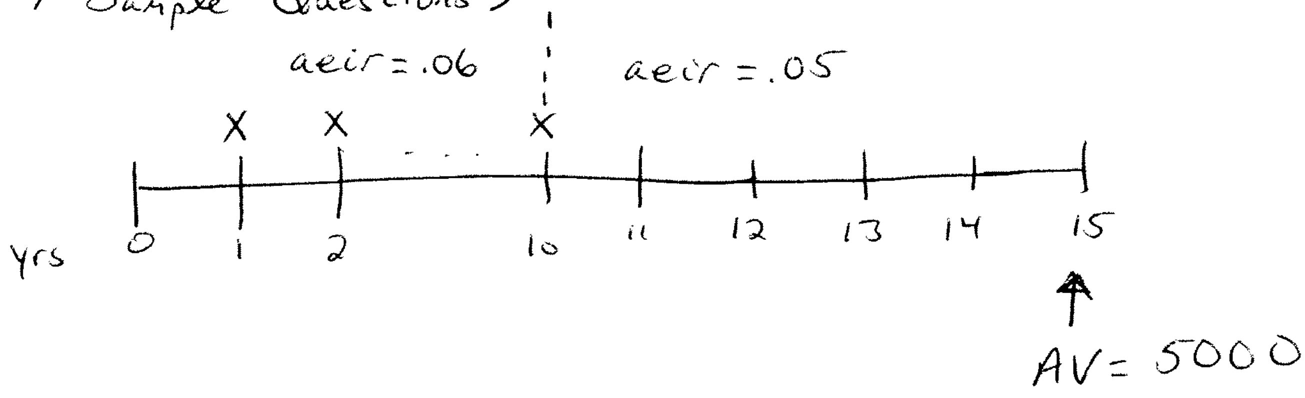


$$PV = Rv + Rv^4 + \cdots \quad (\text{geometric } r = v^3)$$

$$\Longrightarrow \quad 7.21 = \frac{Rv}{1 - v^3} \qquad v = adf \doteq \frac{1}{1.0875}$$

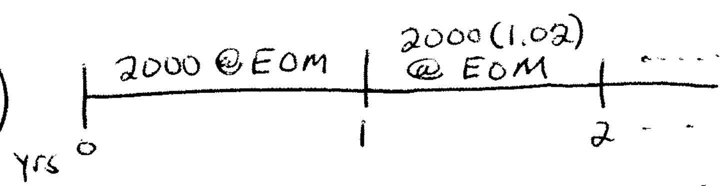
$$\Longrightarrow R \doteq 1.74$$

87/ (Exam FM Sample Questions)



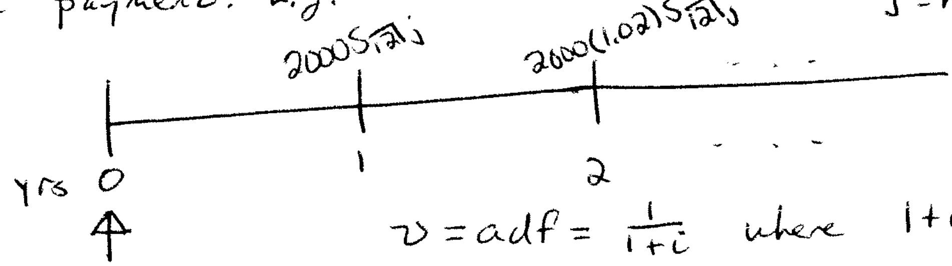
$$AV = 5000 = X\, s_{\overline{10}|.06}\,(1.05)^5 \implies X \doteq 297.22$$

93/ (Exam FM Sample Questions)



Replace monthly payments each year with an equivalent single payment. E.g.

$j = \text{meir} = \frac{.06}{12} = .005$



$v = \text{adf} = \frac{1}{1+i}$ where $1+i = (1.005)^{12}$

$$PV = P \overset{\text{VEP}}{=} 2000\, s_{\overline{12}|j}\, v + 2000(1.02)\, s_{\overline{12}|j}\, v^2 + \cdots \text{ (25 terms)}$$

$$= 2000\, s_{\overline{12}|j} \cdot v_i \left(1 + \frac{1.02}{1+i} + \cdots \text{ (25 terms)}\right)$$

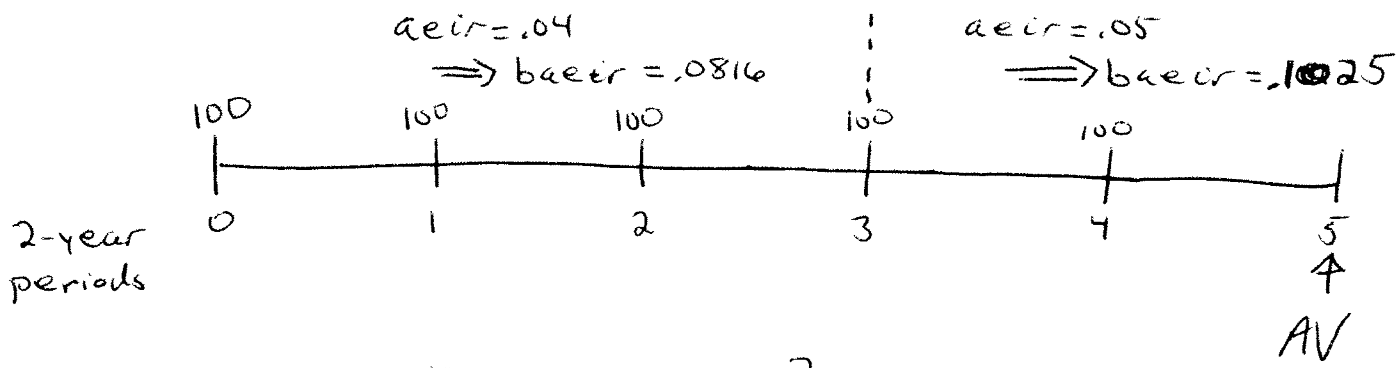
$$= 2000\, s_{\overline{12}|j} \cdot \frac{1}{1+i} \cdot \ddot{a}_{\overline{25}|\left(\frac{1+i}{1.02}-1\right)}$$

See above for values of i & j

$\therefore P = 374{,}443.38$

$\implies$ the lump sum is 56.62 more than P

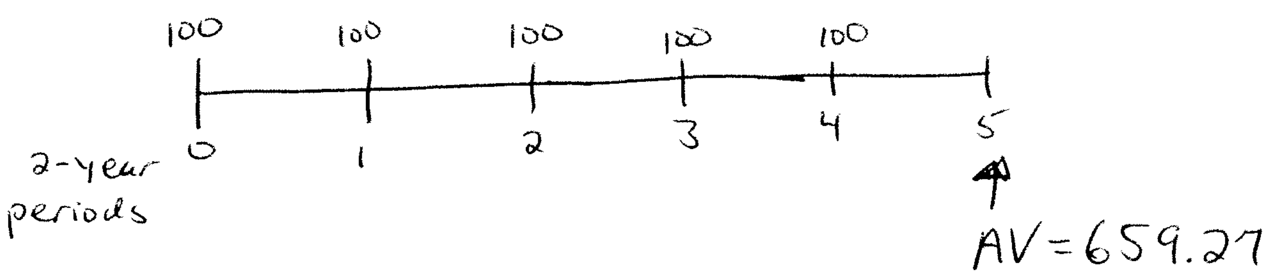
97 / Exam FM Sample Questions) First determine the AV at the end of the 10 years.



$$\therefore AV = 100\,\ddot{s}_{\overline{3}|.0816}(1.1025)^2 + 100\,\ddot{s}_{\overline{2}|.1025}$$

$$\Longrightarrow AV = 659.27$$

Then we seek the aeir for the following timeline



$$\therefore 659.27 = \ddot{s}_{\overline{5}|j} \Longrightarrow j \doteq .0936357 = \text{baeir}$$

$$i = \text{aeir} \Longrightarrow (1+i)^2 = 1+j \Longrightarrow i \doteq .0458$$

98/Exam FM Sample Questions)

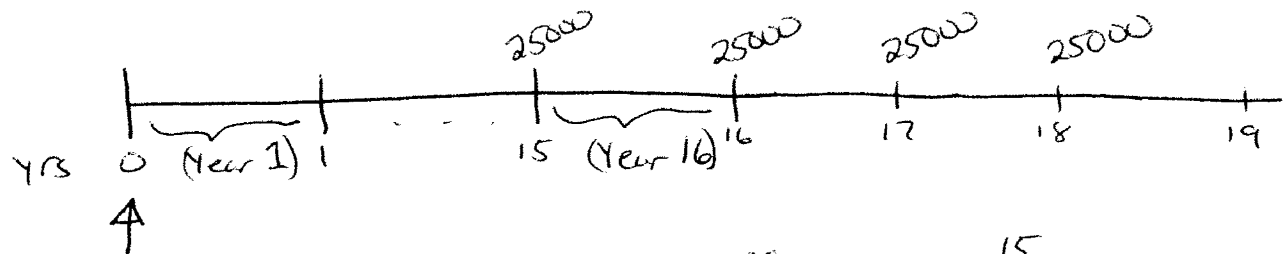
Since compounding, we can use any valuation date

$$PV(\text{deposits}) = X\,\ddot{a}_{\overline{216}|m}$$

$m = \text{meir}$

$(1+m)^{12} = 1.08$

The timeline for withdrawals is:

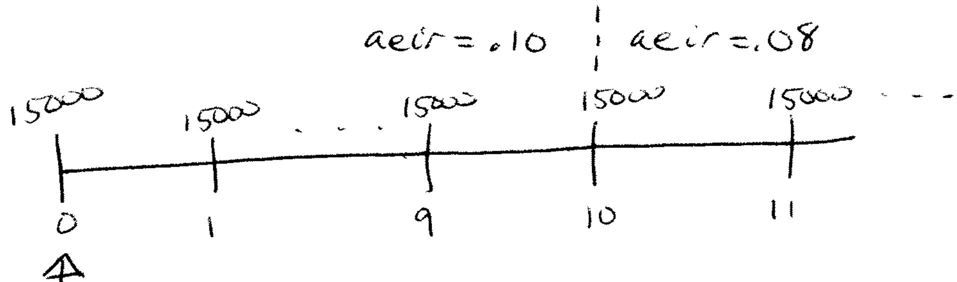


$$\therefore\ PV(\text{withdrawals}) = 25000\,\ddot{a}_{\overline{4}|.08} \cdot v_{.08}^{15}$$

$$\therefore\ X\,\ddot{a}_{\overline{216}|m} = 25000\,\ddot{a}_{\overline{4}|.08}\, v_{.08}^{15}$$

$$\Longrightarrow X \doteq 240.38$$

99 / Exam FM Sample Questions) The timeline for the perpetuity-due is



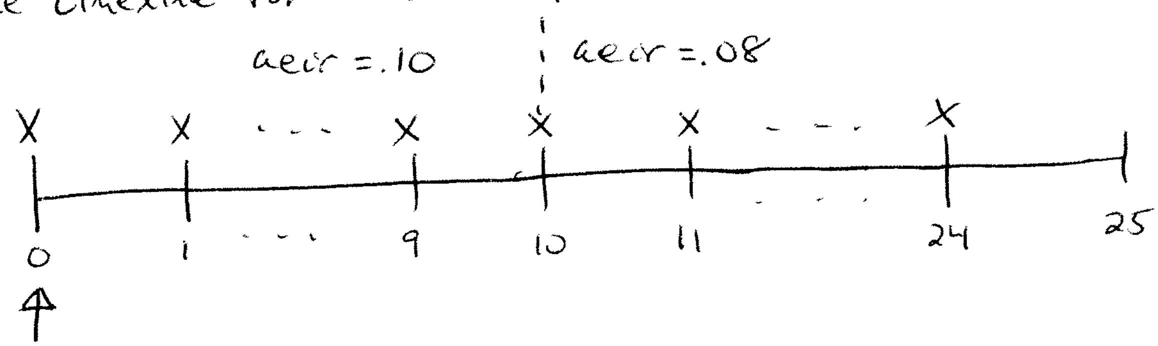
$PV = 15000 \cdot \ddot{a}_{\overline{10}|.1}$ (for the 1st 10 payments)

$+ 15000 v_{.1}^{10} + 15000 v_{.08} v_{.1}^{10} + \cdots$ (for remaining pmts)

$\hookrightarrow$ geometric series with $r = v_{.08}$

$$\therefore PV = 15000\, \ddot{a}_{\overline{10}|.1} + \frac{15000 v_{.1}^{10}}{1 - v_{.08}} \doteq 179{,}457.87$$

Then the timeline for the 25-year annuity-due is



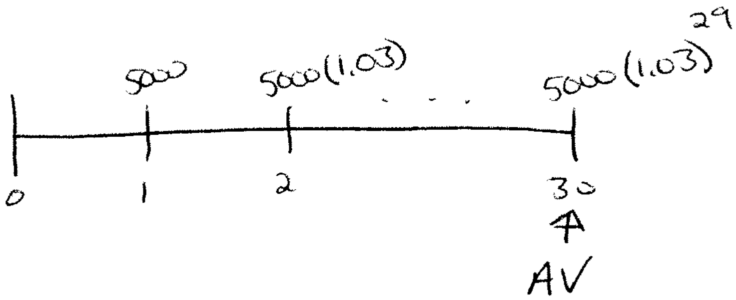
$$PV = X \ddot{a}_{\overline{10}|.1} + X \ddot{a}_{\overline{15}|.08} \cdot v_{.1}^{10} = X\left(\ddot{a}_{\overline{10}|.1} + \ddot{a}_{\overline{15}|.08} \cdot v_{.1}^{10}\right)$$

Setting the two PV's equal and solving for X gives

$$X = \frac{179457.87}{\ddot{a}_{\overline{10}|.1} + \ddot{a}_{\overline{15}|.08} v_{.1}^{10}} \doteq 17384.14$$

102 / Exam FM Sample Questions

First consider deposits:

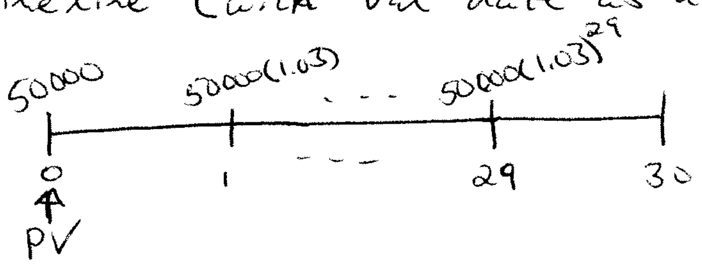


$$AV \overset{VEP}{=} 5000(1.03)^{29} + 5000(1.03)^{28}(1+i) + \cdots \text{ (30 terms)}$$
$$= 5000(1.03)^{29}\left(1 + \frac{1+i}{1.03} + \cdots \text{ (30 terms)}\right)$$

Looking at answer choices, we see that $i > .03$
(Note: we get the same result even if we assume $i < .03$ at this point. We would eventually see that $i > .03$)

$$\therefore AV = 5000(1.03)^{29} \cdot s_{\overline{30|}k} \quad \text{where } k = \frac{1+i}{1.03} - 1$$

For withdrawals, the timeline (with val date as above) is



$$PV \overset{VEP}{=} 50000 + 50000(1.03)\cdot v_i + \cdots \text{ (30 terms)}$$
$$= 50000\left(1 + \frac{1.03}{1+i} + \cdots\right) = 50000\,\ddot{a}_{\overline{30|}k} \qquad k = \text{as above}$$

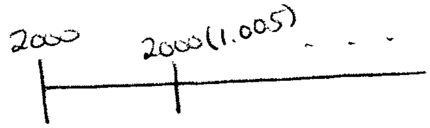
$$\therefore AV = PV \implies \overset{1}{\cancel{5000}}(1.03)^{29}\, s_{\overline{30|}k} = \overset{10}{\cancel{50000}}\,\ddot{a}_{\overline{30|}k}$$

$$\implies (1.03)^{29}\,\frac{(1+k)^{30}-1}{\cancel{k}} = 10\,\frac{1-(1+k)^{-30}}{\cancel{k}}(1+k) \qquad (1+k)^{\pm 30} = \left(\frac{1+i}{1.03}\right)^{\pm 30}$$

$$\therefore \cancel{(1.03)^{29}}\cdot\frac{\cancel{(1+i)^{30}-(1.03)^{30}}}{\cancel{(1.03)^{30}}} = 10\cdot\frac{\cancel{(1+i)^{30}-(1.03)^{30}}}{(1+i)^{\cancel{30}\,29}}\cdot\frac{\cancel{1+i}}{\cancel{1.03}} \implies (1+i)^{29} = 10$$

$\therefore$ Using either expression, $AV = PV = 797836.82$

103 / Exam FM Sample Questions) Since $2010 = 2000(1.005)$, we have



quarters

$v = \text{qdf} = \frac{1}{1+q}$ where $q = \text{qeir}$

$$PV = 100000 \overset{\text{VEP}}{=} 2000 + 2000(1.005)v + \cdots \quad \left(\text{geometric } r = \frac{1.005}{1+q}\right)$$

$$\Longrightarrow 100000 = \frac{2000}{1 - \frac{1.005}{1+q}} \Longrightarrow q \doteq .02551$$

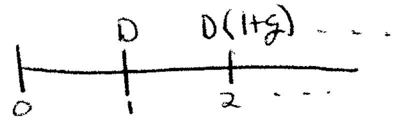
$$i = \text{aeir} \Longrightarrow 1+i = (1+q)^4 \Longrightarrow i \doteq .106$$

125 / Exam FM Sample Questions) Let D = the next dividend for Stock F

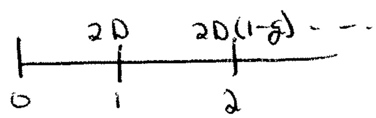
$\therefore$ $2D$ = ________________ J

$\therefore$ the dividend timelines are: (Note: Since we're not told when dividends will be paid, it won't matter. Assume EOY)

Stock F:



Stock J:



$$\text{Value of Stock F} = P^F_{rice} = PV(\text{dividends}) \overset{\text{VEP}}{=} Dv + D(1+g)v + \cdots = \frac{Dv}{1-(1+g)v}$$

$$\text{Value of Stock J} = P^J_{rice} = PV(\text{dividends}) \overset{\text{VEP}}{=} 2Dv + 2D(1-g)v + \cdots = \frac{2Dv}{1-(1-g)v}$$

$$\therefore \frac{\cancel{Dv}\,1}{1-(1+g)v} = 2\left(\frac{2\cancel{Dv}}{1-(1-g)v}\right) \Longrightarrow \frac{1}{1-v-gv} = \frac{4}{1-v+gv}$$

$$\Longrightarrow 1-v+gv = 4(1-v-gv) = 4-4v-4gv$$

$$\Longrightarrow g = \frac{3-3v}{5v} \overset{v=\frac{1}{1.088}}{=\!=} .0528$$

(Remark: Redo, assuming dividends are paid at BOY to see you get the same answer.)